8-1 Sequences

Learning Objectives:

I can define a sequence (arithmetic or geometric) with a formula (recursive or explicit).

I can graph a sequence

I can find the limit of a sequence

Jan 30-8:45 AM

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ is called a sequence

<u>Terms</u>

a_1= 1^{st} term

a_2= 2^{nd} term

a_3= 3^{rd} term

a_n= n^{th} term

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

Finite Sequence

(stops)

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Infinite Sequence

(goes on forever)

Jan 30-8:45 AM

Sequence Formulas

Explicit Formula

$$a_n = \frac{1}{2^{n-1}}$$

$$a_1 = 1$$

$$a_2 = \frac{1}{2}$$

$$a_3 = \frac{1}{4}$$

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots.$$

Recursive Formula

$$a_1 = 1$$

$$a_n = a_{n-1} \cdot \frac{1}{2}$$

Jan 30-8:45 AM

Arithmetic Sequences

You add/subtract the same amount each time

3, 5, 7, 9, 11, ….

Jan 30-8:45 AM

Arithmetic Sequence Formulas

Explicit Formula

$$a_n = a_1 + (n-1)d$$

a_n = nth term
a_1 =1st term
n = # of terms
d = common difference

Recursive Formula

$$a_1 = \#$$

$$a_n = a_{n-1} + d$$

a_n = nth term
a_1 =1st term
n = # of terms
d = common difference

Jan 30-8:45 AM

Ex1. Find

a.) Common difference
b.) Explicit Formula
c.) Recursive Formula

$d = 3$

-2, 1, 4, 7, ...

explicit

$a_n = -2 + (n-1) \cdot 3$

$a_n = -2 + 3n - 3$

$a_n = 3n - 5$

recursive

$a_1 = -2$

$a_n = a_{n-1} + 3$

Jan 30-8:45 AM

Geometric Sequences

You multiply/divide by the same amount each time

3, 6, 12, 24, 48, ...

Geometric Sequence Formulas

Explicit Formula

$$a_n = a_1 \cdot r^{n-1}$$

a_n = nth term
a_1 =1st term
n = # of terms
r = common ratio

Recursive Formula

$$a_1 = \#$$

$$a_n = a_{n-1} \cdot r$$

a_n = nth term
a_1 =1st term
n = # of terms
r = common ratio

Jan 30-8:45 AM

Ex2. Find

a.) Common ratio
b.) Explicit Formula
c.) Recursive Formula

$-\frac{1}{3}, -\frac{2}{3}, -\frac{4}{3}, -\frac{8}{3}, \ldots$

$r = 2$

explicit

$a_n = -\frac{1}{3} \cdot 2^{n-1}$

recursive

$a_1 = -\frac{1}{3}$

$a_n = a_{n-1} \cdot 2$

Jan 30-8:45 AM

Ex3. The 3rd term of a geometric sequence is 16. The 7th term is 4096. Find

a.) Common ratio

b.) The first term

c.) Explicit formula for the nth term

$$a_n = a_1 \cdot r^{n-1}$$

$\div 4$ $\div 4$ $\times r$ $\times r$ $\times r$ $\times r$

1, 4, 16, ___, ___, ___, 4096, ...

$a_1 = 1$

$16 \cdot r^4 = 4096$

$r^4 = 256$

$r = 4$

$a_n = 1 \cdot 4^{n-1}$

$a_n = 4^{n-1}$

Limit

Let ***L*** be a real number. The sequence a_n has limit ***L*** as n approaches infinity, then a_n is said to ***converge to L***. If the terms in the sequence grow unbounded (or do not go anywhere specific), the limit is said to ***diverge*****.**

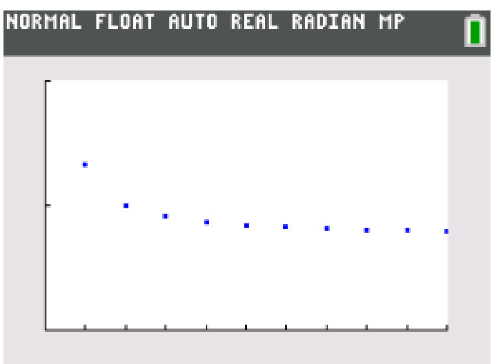
Jan 30-8:45 AM

Ex4. Does this sequence converge or diverge?

$$a_n = \frac{3n+1}{4n-1}$$

$a_n = \frac{4}{3}, 1, \frac{10}{11}, \frac{13}{15}, \frac{16}{19}, \frac{19}{23}, \ldots$

$$\lim_{n\to\infty} \frac{3n+1}{4n-1} = \frac{3}{4}$$



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The Sandwich Theorem for Sequences

If $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$ and $a_n \le b_n \le c_n$ for all n > N, then $\lim_{n\to\infty} b_n = L$

Jan 30-8:45 AM

Ex5. Show that the sequence $a_n = \frac{\sin n}{n}$ converges and find its limit

$a_n = .841, .455, .047, -.189, -.192, -.047, \ldots$

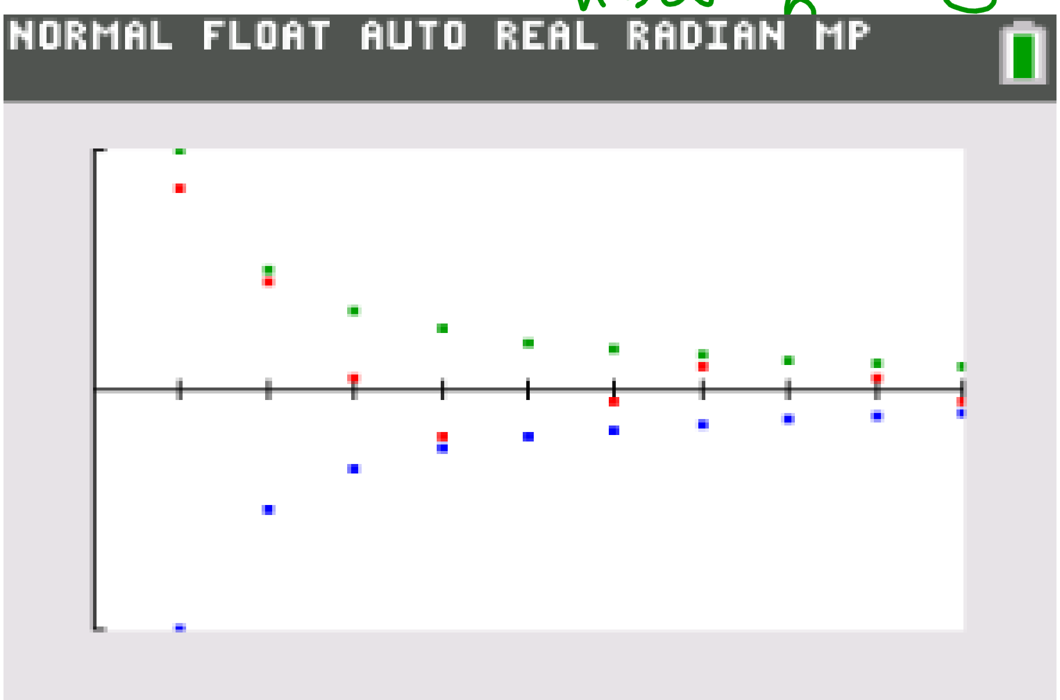
$a_n = \frac{\sin n}{n}$

$-1 \le \sin n \le 1$

$\frac{-1}{n} \le \frac{\sin n}{n} \le \frac{1}{n}$

$\lim_{n\to\infty} \frac{-1}{n} = 0$ $\lim_{n\to\infty} \frac{1}{n} = 0$

$\lim_{n\to\infty} \frac{\sin n}{n} = 0$



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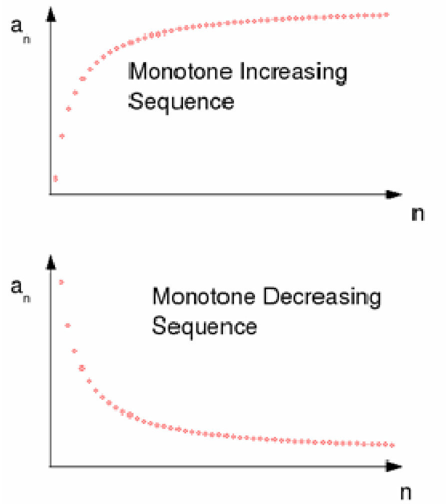
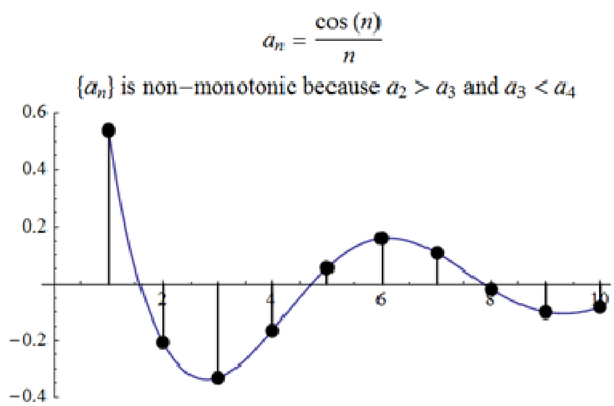
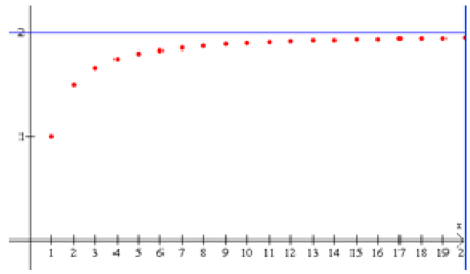
A sequence is **monotonic** if it is either non-decreasing or non-increasing.

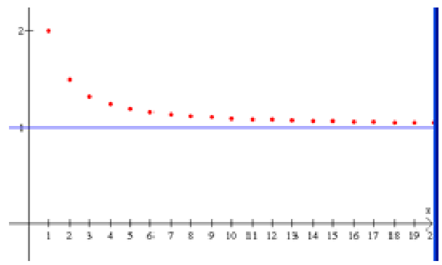




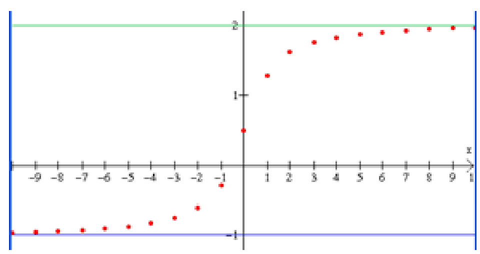
A sequence is **bounded above** if there is a real number M such that $a_n \leq M$ for all n. The number M is called the "upper bound."



A sequence is **bounded below** if there is a real number M such that $a_n \geq M$ for all n. The number M is called the "lower bound."



A sequence is **bounded** if it is both bounded above and bounded below.



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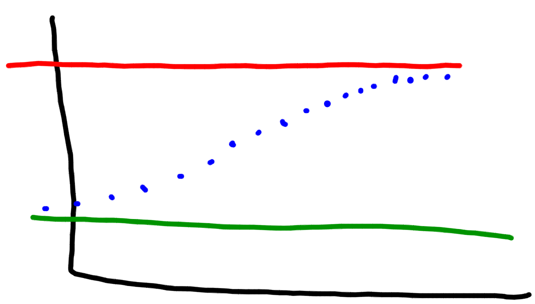
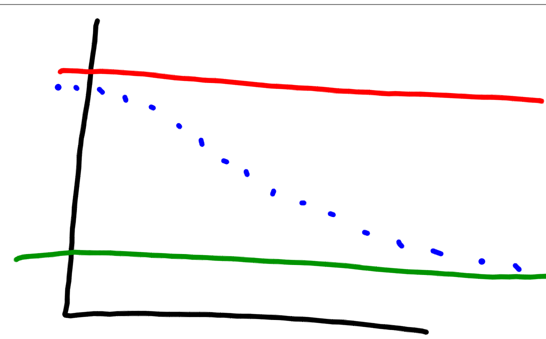
Bounded Monotonic Sequence Theorem

If a sequence $\{a_n\}$ is bounded and monotonic, then it must converge.





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Homework

Pg 441 # 2, 3, 6, 7, 9, 11, 13, 15-17, 23, 25, 27, 31, 33, 34, 35, 37-39, 44-54

Jan 30-8:45 AM